Grade 6 Mathematics, Quarter 1, Unit 1.1

# Compute Using Multidigit Numbers with Finding Common Factors and Multiples

## Overview

**Number of instructional days:** 8 (1 day = 45–60 minutes)

### Content to be learned

- Find Greatest Common Factor of two whole numbers less than or equal to 100.
- Find Least Common Multiple of two whole numbers less than or equal to 12.
- Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
- Use standard algorithm to divide multidigit numbers.
- Use standard algorithm to add multidigit decimals.
- Use standard algorithm to subtract multidigit decimals.
- Use standard algorithm to multiply multidigit decimals.
- Use standard algorithm to divide multidigit decimals.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Develop algorithms for dividing multidigit numbers.
- Understand GCF and LCM using venn diagrams or other models.
- Develop algorithms for all operations to work with decimals.

Attend to precision.

- Understand distributive property and use it to check for correct answers (i.e., reciprocal operations.)
- Fluently utilize a variety of methods to problem solve and check work for correct answers.

### Essential questions

- How could we use estimation to divide two numbers?
- When you divide one number by another number will the quotient always be smaller than the original number? Why or why not?
- How do you use the standard algorithm to divide numbers?
- Which strategies are helpful when adding/subtracting multidigit decimals?
- Which strategies are helpful when multiplying/dividing multidigit decimals?
- When or why would it be helpful to know the GCF/LCM for a set of numbers?
- What strategies are helpful to find the GCF/LCM for a set of numbers?
- How can the distributive property help with computation?

# Written Curriculum

## Common Core State Standards for Mathematical Content

**The Number System 6.NS**

**Compute fluently with multi-digit numbers and find common factors and multiples.**

6. NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

6. NS.2 ~~Fluently~~ divide multi-digit numbers using the standard algorithm.

6. NS.3 ~~Fluently~~ add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

## Common Core Standards for Mathematical Practice

### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

In grade 4, students learned to add, subtract, multiply, and divide two-digit numbers using standard algorithms. Additionally students learned to find all factor pairs for a whole number in the range of 1–100. Students recognized a whole number is a multiple of each of its factors. Students also fluently added and subtracted multidigit whole numbers and could multiply up to 4 digits by 1 digit. In grade 5, students fluently multiplied and divided multidigit whole numbers with decimal to hundredths.

*Current Learning*

Students in grade 6, will develop strategies to identify greatest common factor (GCF) of two whole numbers less than or equal to 100 and the least common multiple (LCM) of two whole numbers less than or equal to 12. Students will use the distributive property to express a sum of two whole numbers. Students will develop standard algorithms for dividing multidigit numbers. Additionally, students fluency in adding, subtracting, multiplying and dividing multidigit decimals will be reinforced.

*Future Learning*

GCF, LCM and distributive property will be used in all algebraic work such as factoring, writing expressions and equations. In grade 7, students will extend all four operations to include positive and negative rational numbers. Additionally, grade 7 students will also convert a rational number to a decimal using long division.

## Additional Findings

According to *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics*, "Children develop an understanding of and fluency with multiplication and division of fractions and decimals by using common procedures to multiply and divide decimals efficiently and accurately."(p. 18)

According to *Adding It Up: Helping Children Learn Mathematics*, "In addition to commutative and associative laws there is a rule, known as the distributive law of multiplication." (p. 76)

According to *Principles and Standards for School Mathematics*, students understand numbers, ways of representing numbers, relationships among numbers, and number systems by using factors, multiples, prime factorization, and relatively prime numbers to solve problems and by working flexibly with decimals to solve problems (p. 214). The book also states that students compute fluently by developing and analyzing algorithms for computing with decimals (p. 214). Additionally, "Multiplying and dividing fractions and decimals can be challenging for many students because of problems that are primarily conceptual rather than procedural." (p. 218) Furthermore, "Tasks, such as the following, involving factors, multiples, prime numbers, and divisibility, can afford opportunities for problem solving and reasoning." (p. 217)

Grade 6 Mathematics, Quarter 1, Unit 1.2

# Composing and Decomposing to Find the Area and Surface Area of Polygons

## Overview

**Number of instructional days:** 7 (1 day = 45–60 minutes)

### Content to be learned

- Represent 3-dimensional figures using nets made up of rectangles and triangles.
- Use nets to find surface area of prisms and pyramids.
- Apply nets and surface area to solve real-world problems.
- Find the area of triangles, special quadrilaterals, and polygons by composing or decomposing into other shapes.
- Solve real-world problems involving area of polygons.

### Mathematical practices to be integrated

Model with mathematics.

- Use nets to find surface area of 3-dimensional figures.
- Create nets to form 3-dimensional figures.
- Find the area of figures by using various mathematical manipulatives.

Attend to precision.

- Find the area and/or surface area of 2- and 3-dimensional figures and shapes.
- Utilize a variety of methods to find area and/or surface area of shapes and check for correct answers.

### Essential questions

- How do you represent a net by constructing a three-dimensional figure made up of rectangles and triangles?
- How do you compose and decompose polygons to create other figures?
- What does it mean to compose a figure into rectangles?
- How can the net of any 3-dimensional figure help calculate the surface area of that shape?
- What real-world situations require knowledge of surface area?
- How can the net of any 3-dimensional figure help calculate the surface area?
- How do you find the area of irregular shapes?
- What real-world situations require finding area?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| Geometry | 6.G |
|---|---|

**Solve real-world and mathematical problems involving area, surface area, and volume.**

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

## Common Core Standards for Mathematical Practice

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

In grade 3, students learned to measure area by counting unit squares and related area to operations of multiplication and division. In grade 5, students learned to classify two-dimensional figures in a hierarchy based on properties.

*Current Learning*

In grade 6, students represent three-dimensional figures using nets made up of rectangles and triangles and use these nets to find surface area. Students use composing and decomposing strategies to find the area of right and other triangles, special quadrilaterals, and polygons.

*Future Learning*

In grade 7, students will describe the two-dimensional figures that result from slicing three-dimensional figures. They will solve real-world and mathematical problems involving area, volume, and/or surface area of two- and three-dimensional objects.

## Additional Findings

According to *Principles and Standards for School Mathematics*, "Precisely describe, classify, and understand relationships among types of two and three dimensional objects using their defining properties. Additionally, frequent experiences in measuring surface area can also help students develop sound understandings of the relationships among attributes and of the units appropriate for measuring them. (p. 232)

According to *A Research Companion to Principles and Standards for School Mathematics*, "Geometry is the forgotten strand of mathematics that offers us a way to interpret and reflect on our physical environment and can serve as a tool for studying other topics in mathematics." (p. 171)

### Grade 6 Mathematics, Quarter 1, Unit 1.3

# Using Models and Equations to Interpret and Compute Fractions

## Overview

**Number of instructional days:** 15 (1 day = 45–60 minutes)

### Content to be learned

- Use equations to interpret and compute quotients of fractions.
- Use fraction models to interpret and compute quotients and fractions.
- Use fraction models to solve real-world problems for division of fraction by fraction.
- Use equations to solve real-world problems for division of fraction by fraction.

### Mathematical practices to be integrated

Model with mathematics.

- Models to divide fractions.
- Critique peer models.

Use appropriate tools strategically.

- Rationalize the tools they select for solving problems.
- Select appropriate tools to model division of a fraction by a fraction.
- Detect errors by strategically using estimation and other mathematical knowledge.

### Essential questions

- How you model division of fractions?
- How do you divide a fraction by a fraction using equations?
- What is a story that demonstrates division of fractions?
- What does it mean to divide a fraction by a fraction?
- When you divide a fraction by a lesser fraction, what do you expect your quotient to be?
- What algorithm makes sense for dividing a fraction by any fraction?

# Written Curriculum

## Common Core State Standards for Mathematical Content

**The Number System 6.NS**

### Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

## Common Core Standards for Mathematical Practice

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Clarifying the Standards

*Prior Learning*

In grade 3, students developed an understanding of fractions by using fraction models to represent parts of a whole. In grade 4, students developed an understanding of fraction equivalence. They become fluent with addition and subtraction of fractions with like denominators. In grade 5, students developed fluency with addition and subtraction of fractions and developed an understanding of multiplication of fractions and division of fractions in limited ways.

*Current Learning*

In grade 6, students complete the understanding of division of fractions. Students use the meaning of fractions and the meaning of multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students expand the scope of problems for which they can use multiplication and division to solve problems.

*Future Learning*

Students will develop an understanding of operations with rational numbers and work with expressions and linear equations.

## Additional Findings

According to *Principles and Standards for School Mathematics*: "Students will deepen their understanding of fractions and they should become proficient in using them to solve problems." (p. 215)

According to *Curriculum Focal Points*, "Students expand their repertoire of problems that they can solve by using multiplication and division and by building on understanding of fractions." (p. 18)

Grade 6 Mathematics, Quarter 1, Unit 1.4

# Reading, Writing, and Evaluating Algebraic Expressions

## Overview

**Number of instructional days:** 10 (1 day = 45–60 minutes)

### Content to be learned

- Write numerical expressions involving whole number exponents.
- Evaluate numerical expressions involving whole number exponents.
- Read and write expressions using letters to stand for numbers.
- Identify parts of an expression using mathematical terms.
- Use variables to represent expressions in real world problems.
- Understand variables can represent any unknown number or specific set.

### Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Understand the use of substituting letters for any numeric quantity in an expression.
- Understand that numeric quantities represent real world problem and their solution.
- Understand that an expression can be a single entity or the sum of two terms, for example, 2(8 + 7)**.**

Attend to precision.

- Solve for an unknown using the basic order of operations.
- Clarify the meaning of symbols used in expressions.
- Use math language and appropriate vocabulary to describe expressions.
- Read, write, and evaluate unknown variables using mathematical terms.

### Essential questions

- How do you evaluate a numerical expression involving exponents?
- How do you write an expression using variables to represent a real-world situation?
- What are the mathematical terms that name the parts of an expression?
- What is an example where a variable is represented by a non-numeric expression that applies to a real-world situation?
- How do you evaluate an expression in mathematical and real-world problems when given a value for the variable?

# Written Curriculum

**Common Core State Standards for Mathematical Content**

**Expressions and Equations 6.EE**

**Apply and extend previous understandings of arithmetic to algebraic expressions.**

6. EE.1 Write and evaluate numerical expressions involving whole-number exponents.

6. EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

**Reason about and solve one-variable equations and inequalities.**

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

**Common Core Standards for Mathematical Practice**

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

In third grade, students began working with order of operations. In grade 4, students were introduced to multiplication equations as a comparison. In grade 5, students learned parentheses, brackets, or braces as part of mathematical expressions. They wrote simple expressions that record calculations with numbers and interpret numerical expressions.

*Current Learning*

Students develop an understanding of whole-number exponents and their use in numerical expressions. Students develop an understanding of whole-number operations so that they can solve for unknown variables represented by alphabetic letters. Students are becoming fluent with mathematical terms for solving equations. Real-world situations are converted into algebraic expressions.

*Future Learning*

In grade 7, students will solve numeric and algebraic expressions, equations and inequalities with rational numbers applying the properties of operations. In grade 8, students will work with integer exponents. Students will use variables in a real-world or mathematical problem and inequalities to solve problems by reasoning about the quantities. Students will apply properties of operations and eventually use this knowledge in linier and quadratic equations.

## Additional Findings

According to *Curriculum Focal Points*: "Students understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately." Additionally, students should develop an initial conceptual understanding of different uses of variables. (p. 18)

According to *Adding It Up; Helping Children Learn Mathematics*: "By emphasizing both the relationships among quantities and ways of representing these relationships instruction can introduce students to the basic ideas of algebra as a generalization of arithmetic." (p. 419)

According to *Principles and Standards for School Mathematics*, students should develop an initial understanding of several different meanings and uses of variables through representing quantities in a variety of problem situations. In the middle grades, students should also learn to recognize and generate equivalent expressions and solve linear equations. (p. 223)